Evaluation by a teacher of the suitability of her mathematics class[1]*

Yuri Morales-López[2]
http://orcid.org/0000-0002-2973-4038
Vicenç Font Moll[3]
http://orcid.org/0000-0003-1405-0458

Abstract

Evaluation of teaching practice is a necessary task to improve the teaching and learning processes. The purpose of this study is to analyze the assessments made by a Mathematics in-service teacher when she observed a video recording of a class in which she taught students about the logarithmic function in the fourth year of Secondary Education. To do so, a case study with non-participant observation was carried out. The research design is a case study using an Onto-semiotic approach to mathematical knowledge and instruction. Two research protocols were applied: 1) an expert analysis based on this approach to determine mathematical practices, processes and objects, and 2) an evaluation of didactic suitability. The assessments made were classified using the criteria, components and indicators of didactic suitability proposed in the Didactic-Mathematical Knowledge and Competencies (DMKC) model. The results show that the teacher's evaluations emphasize epistemic and interactional criteria and that such evaluations can be classified and organized using didactic suitability criteria, and their components and indicators. In addition, the methodology applied is useful for teachers who wish to evaluate the mathematical instruction they provide, and can allow them to recognize elements and topics for improvement in their educational practice.

Keywords

DMKC model – In-service teachers – Self-assessment – Logarithms – Video recordings.

Introduction

One of the principal tasks associated with instruction processes is to evaluate what happens in the classroom and try to improve it. This task depends on multiple factors, and when thinking about evaluating these processes, several questions are fundamental: What parameters should be considered to assess the relevance of a teaching and learning process? What is a relevant, appropriate, or excellent class? What capacity for identification and interpretation does the person who assesses classroom situations have? What are the beliefs about a good math class?

Although these questions can give rise to broad research agendas in the area of mathematics education, one way to approach the study of the evaluation of instructional processes is from the didactic analysis model proposed by the Onto-semiotic Approach of mathematical knowledge and instruction (GODINO; BATANERO; FONT, 2007), where the final purpose of the analysis is to propose a possible improvement of a training process that is already being implemented.

Evaluating the implementation of a class is not easy if there are no criteria to guide such evaluations. One way to approach the problem of how to assess an instructional process is to look for desirable and undesirable characteristics in a class, seeking to determine the suitability of these characteristics in a given context. At one extreme this could be understood as compliance with certain static and predefined indicators which have to be achieved to obtain a class with desired characteristics, to isolate each variable and obtain a score with a given metric. Another approach that can be used is to identify certain indicators, how they influence each other and how they are interrelated in a given context.

With reference to this way of conceiving of these two points of view, Huang and Li point out that, "[...] notions about effective instruction in mathematics classes and the ways to achieve them are culturally specific and mutually affect each other within a particular context (specific content and a group of persons)" (2009, p. 308). From this point of view, excellence may be seen as a dynamic balance between characteristics that are defined as correct and their implementation in a particular context (HUANG; LI, 2009).

In this sense, the Onto-semiotic Approach (OSA) (GODINO; BATANERO; FONT, 2007) and, in particular, the Didactic-Mathematical Knowledge and Competencies model (DMKC) (GODINO et al., 2017; BREDA; FONT; PINO-FAN, 2017) offer theoretical tools, especially the notion of didactic suitability, which is specified in six criteria, which in turn consist of interrelated components and indicators whose weights are relative to changing temporal and contextual circumstances (GODINO; BATANERO; FONT, 2007, BREDA; FONT; PINO-FAN, 2018).

In the specific case of Costa Rica, secondary school mathematics teachers have multiple shortcomings in both pedagogical and didactic-mathematical knowledge (MORALES-LÓPEZ, 2017). There is also evidence that a similar situation may occur in some stages of the initial training of these teachers (MORALES-LÓPEZ, 2017; MORALES; FONT, 2017). In this situation, it is essential to rethink topics such as initial training,

professional development and teacher recruitment (a macro-scale view), and that in-service teachers assess and propose improvements to their own work (a microscale vision).

The research presented here is concerned with the main elements of analysis and assessment that an in-service teacher uses when she is asked to comment on a class she taught and that was recorded on video, without any previously established guidelines for analysis. The goal of this activity was to determine which factors the teacher considered to be of interest and her assessment of each situation. To accomplish this, a class was recorded on video in which the teacher was teaching the logarithmic function and was afterwards asked to observe the video and indicate what elements were of interest for a possible analysis of her math class and why they interested her.

The novelty of this approach is that this didactic analysis, the assessment of didactic suitability, is taken as a reference to study the analysis and assessment made by the teacher about the recording, when she reflects on the class she taught. The information that this research can generate provides tools for a teacher to evaluate his or her mathematics class in a context and can also provide information on how teachers identify situations of interest in their classes.

Theoretical framework

Reflections on what happens in a mathematics class and video recordings as resources

The ability to assess what happens in a classroom seems to be one of the basic skills that a mathematics teacher must have. It is therefore necessary to have conceptual frameworks and appropriate methodological tools (GODINO; BATANERO, 2009). For Seckel, a teacher expresses or demonstrates the competence of reflection on teaching practice, his own or someone else's if "[...] he or she critically analyzes their pedagogical practice and that of other teachers in terms of its impact on student learning and proposes and provides a basis for changes to improve it" (2016, p. 40). This research seeks to define possible strategies that lead to such an assessment, so that the teacher can notice or identify situations of interest and explain them to make sense of alternative practices (MASON, 2002, 2017).

Lesson Study (CLARKE et al., 2006; ISODA et al., 2007; LEWIS; TSUCHIDA, 1998; YOSHIDA, 1999) is a strategy or methodology used for the improvement of mathematics education which has attracted interest in the study of processes of initial training and professional development of in-service mathematics teachers.

Although *Lesson Study* can be used to create activities or good practices, Yoshida states that this is not his goal. His main concern is that teachers who are outside the context of study or outside of school communities who study lessons "[...] adopt without any reflection these practices in which there are gaps in their understanding, due to the lack of mathematical and pedagogical knowledge , and of skills necessary to observe and evaluate the learning of their students in the classroom" (2012, p. 141). Although this

author focuses on the case of teachers in the US, this concern is valid in a large part of the educational community.

The use of videos to analyze instructional processes is also a strategy currently used to evaluate the abilities and competencies of a teacher based on selected situations (BORKO et al., 2011; KAISER et al., 2015).

Kaiser and other authors (2015) state that this strategy is important because it provides a broader picture of situations in the classroom and allows better assessment of the capacity and knowledge of the teachers who are evaluated (as compared, for example, to evaluation of situations described using paper and a pencil). These authors also warn that there could be an important limitation on this strategy when the classes recorded are artificially constructed (imposed situations) because the actions that actually occur in a real class are lost. There are already several theoretical frameworks that support the possible uses of videos of mathematics teachers depending on the purposes for which they are used (COLES, 2014).

Providing theoretical support to the benefit of video analysis goes beyond the objective of this article. In this research, the teacher analyzes his or her own mathematical instruction in the classroom, assessing their work and analyzing recorded videos, situations, decisions or behaviors that were made consciously or unconsciously.

Given the international trends mentioned previously, the use of video to study real situations in instruction has attracted increasing attention and some research provides specific results on teacher training and video recording (ROSAEN et al., 2008; KLEINKNECHT; SCHNEIDER, 2013).

The DMKC model

The didactic analysis of classroom activities can be approached from different perspectives. In particular, the DMKC model (GODINO et al., 2017; BREDA; FONT; PINO-FAN, 2017) based on the Onto-semiotic Approach (OSA) (GODINO; BATANERO; FONT, 2007) offers theoretical tools to structure this analysis.

The DMKC model considers that one of the key skills of the mathematics teacher is the general competence of analysis and didactic intervention, which is broken down into five sub competences linked to the analysis of: a) global meanings, b) mathematical practices, c) didactic configurations, d) standards, and e) didactic suitability. This model is used in the present study, with an emphasis on the assessment of the didactic suitability sub competence. The didactic suitability of the OSA is viewed as a coherent articulation of six specific suitability criteria (Table 1).

Each of these criteria can be broken down into components and indicators as a category, in order to make them operational. For this research, Font's (2015) proposal is used. The criteria and components of didactic suitability of the aforementioned proposal are detailed below (due to space issues, the indicators are not detailed) (Table 2).

Table 1- Criteria of didactic suitability

Criterion	Description
Epistemic	To assess whether the mathematics taught is "good mathematics".
Cognitive	To assess, before beginning the instructional process, if what is to be taught is reasonably distant from what students know and, after the process is completed, if the learning achieved approaches that which was intended to be taught.
Interactional	To assess if the interaction has resolved doubts and difficulties of the students.
Mediational	To assess the adequacy of material and temporary resources used in the training process.
Affective [or emotional]	To assess the level of involvement (interest, motivation) of students in the instructional process.
Ecological	To assess the adaptation of the instructional process to the educational project of the center, curricular guidelines, conditions of the social and professional environment, etc.

Source: Based on Font, Planas and Godino (2010, p. 102).

Table 2- Criteria and components of suitability

Criterion	Component
Epistemic	(ES1) Errors, (ES2) Ambiguities, (ES3) Richness of processes, (ES4) Representativity
Cognitive	(CS1) Previous knowledge, (CS2) Curricular adaptation to individual differences, (CS3) Learning, (CS4) High cognitive demand
Interactional	(IS1) Teacher-student interaction, (IS2) Interaction between students, (IS3) Autonomy, (IS4) Formative evaluation
Mediational	(MS1) Material resources, (MS2) Number of students, schedule and classroom conditions, (MS3) Time
Afective or motional	(AS1) Interests and needs, AS2) Attitudes, (AS3) Emotions
Ecological innovation	(CIES1) Adaptation to the curriculum, (CIES2) Intra- and interdisciplinary connections, (CIES3) Social and work utility, (CIES4) Didactic innovation

Source: Based on FONT (2015).

The methodology used in this study is described in the next section.

Methodological framework

This is a qualitative case study, where the characteristics of a few individuals are studied in detail (STAKE, 1995). A video of a class recorded in 2015 was used. The study was conducted in 2016 and is descriptive, involving non-participant observation.

Description of the participant

A 38-year-old Secondary Education math teacher with a Bachelor's Degree in Mathematics Teaching from a state university in Costa Rica in 2008 was selected. By

2014, she had 14 years of teaching experience (she began working before graduating). In 2010 she began working as a teacher at the school where she was recorded. The selection of the participant was based on convenience (STAKE, 1995).

Protocol

A recording of a math class of a teacher was made as she taught the logarithmic function in a public secondary school in Costa Rica. In the first stage, the recording (1 hour, 5 minutes, and 40 seconds long) was examined as follows:

1. An expert analysis of the first four types of didactic analyses proposed in the didactic analysis model based on the OSA constructs was carried out following Godino, Contreras and Font (2006) and Pochulu and Font (2011), to identify mathematical practices, objects and processes, teacher and student functions, didactic configurations, semiotic conflicts, patterns and norms.

2. With the information obtained from Point 1, an expert assessment of didactic suitability was carried out (the fifth type of didactic analysis of the didactic analysis model based on OSA constructs) to be used as a reference for the study of the teacher's own evaluations (see Montiel and other authors, 2009, as a model of the study of valuations).

In a second stage, the teacher who taught the class was asked to analyze her activities to determine her explicit intentions and assess the situations that occurred. To do so, a workshop was held with the teacher in which the video was shown on two consecutive occasions; afterwards, she was asked to record in writing the elements of the recorded mathematical activity that she considered to be important to analyze (after the two continuous showings she was allowed to control the presentation of the video in additional showings using forward, delay, pause or stop). The activity lasted 3 hours and 30 minutes.

In a third stage, the teacher's ratings were classified using the criteria and components of didactic suitability described in the section on the DMKC model (FONT, 2015).

Finally, in a fourth stage, the previously categorized information about the teacher's observations (third stage) and the analyses carried out in the first two stages were contrasted to show which assessments coincided with the expert assessment of the criteria, components and indicators of suitability, and which were not identified or evaluated by the teacher; the content of the teacher's evaluation was analyzed to determine its relevance in her justification.

Context and description of the class

The teacher worked with her students on the concept of the logarithmic function, which is presented in the fourth year of high school as specified in the Transition Plan between the previous and current Programs of Studies (COSTA RICA, 2012). The students were between 15 and 17 years old. She organized the class based on the specific suggestions contained in the Mathematics Studies Program of the Ministry of Public Education of

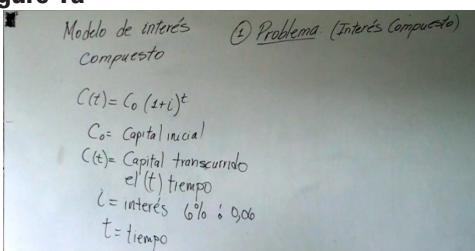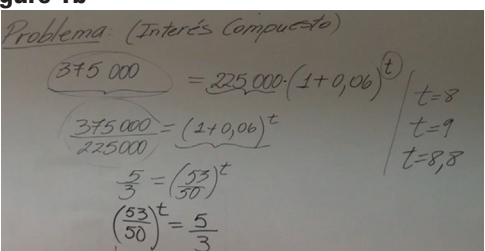Costa Rica (COSTA RICA, 2012). Table 1 describes the instructions of the Mathematics Studies Program.

Table 1 - Knowledge, specific skills and specific instructions described in the official Mathematics Program - Costa Rica
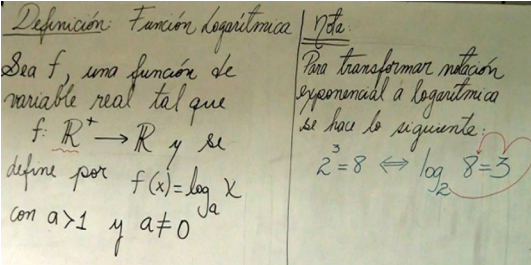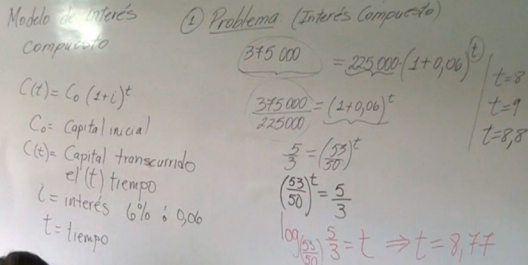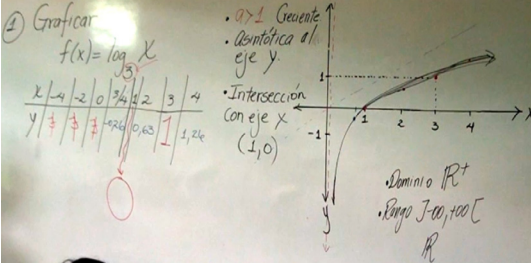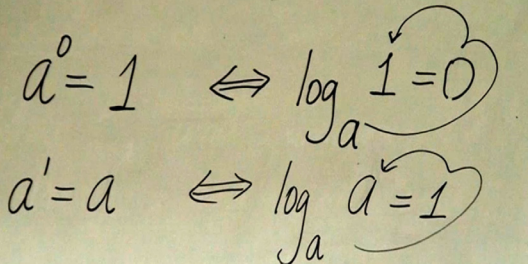
Knowledge: Logarithmic functions, the function $\log_a x$, and logarithmic equations
Specific skills   9. Identify the logarithmic function as the inverse of the exponential function.   10. Graphically and algebraically analyze logarithmic functions.
Specific instructions   •   To introduce the topic, it is useful to present a problem in which the logarithmic function arises naturally.   (Problem) Laura Marcela deposits 225,000 colones in her savings account in a bank. and at the end of t years receives a notification from the bank indicating that her account has 375,000 colones. If the interest rate is 6% compounded monthly and if she did not make a new deposit or withdrawal during those years, how many years have passed between the deposit and the bank's notification?   In this problem it is important to present the model $$C(t) = C_0 \left(1 + i/n\right)^t$$ and based on it to justify the need to introduce the logarithmic function.   •   During the closing stages, introduce the logarithmic function as the inverse of the exponential function. Use the composition of functions to justify this property.   Each student should know that if $a > 0$, $a \neq 1$, then $\log_a y = x$ if and only if $y = a^x$. Therefore, $\log_a y$, $y > 0$ is the number to which base a must be raised to obtain y.   •   Change from the exponential to the logarithmic form and from the logarithmic to the exponential form.

Source: Transcript of the Ministry of Public Education (COSTA RICA, 2012, p. 415-416)

Figures 1a, 1b, 1c, 1d, 1e, 1f, show the sequence of work used by the teacher in her class.

Figures 1a, 1b, 1c, 1d, 1e, 1f -. Sequence of the notes on the whiteboard

Figure 1a

Compound interest model
Problem (compound interest)
Initial capital
Capital accumulated at time (t)

Figure 1b

Problem (compound interest)

Figura 1c	**Figura 1d**
Definition: logarithmic function Let f be a function of a real variable such that f: IR+ -> IR and it is defined by f(x) = logax where a>1 and a≠0 Note:To transform exponential notation to logarithmic notation, do the following:	See text from Figure 1.
Figure 1e	**Figure 1f**
Graph Increasing Asymptotic to the y axis Intersection with the x axis Domain: IR+ Range:]-∞, ∞[, IR	

Source: Photographs taken from the video recording

In the first part (Figure 1a), the teacher reminds students of the problem raised in the previous class and writes the mathematical model that solves that problem. She defines the names of parameters of the model that she indicates will serve to resolve the problem that she has presented and asks students to form work groups. When starting work after the question, she tells them to use the trial and error method. In Figure 1b, students mention some approximations. A group of students indicated that the value was between t = 8 and t = 9. She states that it is obvious that they do not have tools to solve the exponential equation, so she is going to introduce them to the logarithmic function to solve the problem.

Figure 1c shows that the teacher wrote an incorrect definition of the logarithmic function, and the way to convert an exponential expression to a logarithmic expression.

She explains the definition and introduces the procedure to convert them. She does this using a specific example ($2^3 = 8$ if and only if $\log_2 8 = 3$) and uses an associative memory technique based on a diagram whose shape is similar to that of a heart. Figure 1d shows how she returned to the previous problem and used the note explaining the conversion using the heart technique (a heart shape as a graphical mnemonic rule). The teacher asks them to indicate where to put the data in the model written on the board to perform the conversion of $(53/50)^t = (5/3) \Leftrightarrow t = \log_{(53/50)} (5/3)$. She then says that they should enter the information in their calculators.

In the penultimate part (Figure 1e) the teacher graphs a logarithmic function [$f(x) = \log_3 x$]; to do this, she writes a table on the whiteboard with the values of the variable x and ask them to complete the table using the calculator (the values of the dependent variable y). She deliberately includes values not included in the R^+ domain so that they can see what happens using the calculator. Next, the teacher talks about the positive values of the table she wrote on the board and asks them to add a 1 in the table (in the domain), because she forgot to do so (her intention was to talk about the intersection with the X axis later). She comments on the characteristics of the values of the dependent variable y (which have an increasing trend). She then draws a line on the Cartesian plane, mentions that the line does not touch the Y-axis, and reminds them of the meaning of an asymptote (the analogy she used was that there was an electrified fence and the line could not touch it).

She then tells the students that the intersection with the X-axis is (1,0) and asks them about the intersection with the X-axis of the graph of the exponential function. She discusses the injective and surjective nature of the function and indicates that the exponential function is the inverse function of the logarithmic function; next, she graphs $g(x) = x$ and reminds students that the inverse functions, graphically, look like a reflection with respect to the y-axis, and asks them for the name of the reflected line. Since no one answers her, she tells them that it is the graph of the exponential function.

In Figure 1f, the teacher uses the property that $a^0 = 1$ and the heart technique to infer that $\log_a 1 = 0$. She then relates this to the value of the table that she had previously forgotten to write and says that this is always true. In the same way, she generalizes this for $a1 = a$ if and only if $\log_a 1 = a$. Finally, she tells them that in the next class they will plot $f(x) = \log_{(3/4)} (x)$.

Discussion of results

In the first stage (1. An analysis of didactic configurations in the instructional processes), a description of the didactic configurations of the instructional process recorded in the video was carried out, based on the model described for this purpose in Godino, Contreras and Font (2006). The mathematical objects present, the processes carried out, the role or function of the teacher in differing stages, the didactic configuration, semiotic conflicts (when there were any), patterns, interactions, and the rules shown in the video were identified.

For the second part of stage 1 and later stages, the evaluations made by the teacher of her own class were presented and analyzed using the notion of didactic suitability. The indicators that the teacher has indicated are important to study are compared to those identified in the analysis of the first stage, to determine which elements the teacher identified in her assessment and which of those that appear in the video she did not identify. In the case of the elements that do appear in her assessment, we want to know if it is consistent with the estimate in stage 1 described in the methodology.

Elements of suitability present in the teacher's evaluation of the instructional process

The components and indicators of didactic suitability criteria for the class taught by the teacher were analyzed in two ways. On the one hand, the elements that the teacher used when she made her assessment were discussed. On the other hand, the elements that she did not use in her analysis and that could be of interest for the analysis and assessment of her teaching (as indicated in the expert assessment) were also mentioned.

Components and indicators of epistemic suitability [ES]

[ES1] an erroneous definition of the logarithm. With respect to this component, the teacher indicated that possibly because she was nervous, she made an error when defining the logarithmic function with a base different from 0 and greater than 1. It should be noted that this error caused several problems, such as when a student tried to use the definition to justify the domain when graphing $f(x) = log_3 x$. In addition, the teacher failed to notice that she made an error in the use of the concepts of approximation and exact values; she even argued that the value obtained using a calculator is the exact one, neglecting or devaluing the approximations that she made in the earlier part of the class. This caused several students to care more about rounding decimals than about what they were solving.

[ES2a] Regarding possible ambiguities, the teacher's assessment noted that she tried to make a didactic transposition that was perhaps inadequate, when she stated that the class members would test t values using the calculator to find an approximation. **[ES2b]** She also noted that she has never used the symbol "does not exist" ($\nexists$) in her class and indicated that she did not remember that she needed it until that moment, and did not define it correctly. Another ambiguity that she detected is the analogy of an "electrified fence" when discussing the asymptote; she acknowledged that this analogy might be inadequate, although she indicated that she would use it again **[ES2c]**. The problem with this metaphorical analogy is that it did not convey everything that she wanted to, and every time she referred to the asymptote, she had to use further analogies or metaphors (like "approaching, but not touching") to convey her idea. (FONT; BOLITE; ACEVEDO, 2010, FONT et al., 2010).

An ambiguity that the teacher did not identify explicitly was that which determined the choice of the values of the table and the graph that she drew, since it allowed, at least visually, a logarithmic adjustment, but also seemed to be linear; even a student

noticed this (when the teacher asked a question, a student stated that the graph was a straight line). With reference to this problem, she only indicated in her written notes that "I should have used a more exact scale (I do not like to graph)" **[ES2d]**. Another element that the teacher did not mention is that, at various times, she talked about the model function that they were working with as if it were an exponential function, which is not at all rigorous. The correct way to discuss it is to consider it as a particular case of an equation with parameters.

[ES3a] Regarding the richness of processes, the teacher indicated that at certain times more effort might have been made, such as assisting students to understand the use of 6% or 0.06. She was aware that she explicitly instructed the students to solve the equation "using an algebraic strategy", but noted that they responded by indicating what actions could be taken to simplify the equation to the form $a = b^x$ that she intended **[ES3b]**. Another aspect that she indicated that she did not take advantage of was the discussion that could have been had about converting an expression $t = 8.77$ years to a more common expression like years, months and days **[ES3c]**. The teacher evaluated as "great" the fact that they made the generalization that in the table "only the negatives" did not work **[ES3d]**.

There were several elements that she did not identify. The exposition of the problem was linked to a function model which initially suggested to the students the method they should use to find a solution (trial and error); this made the problem a very simple exercise and limited the richness of the process. It is important to point out that the introduction of the topic began with a model to be applied, when it might have been more useful to begin with a problem and finding a way to model it. In addition, the important processes of regulation were carried out completely by the teacher, without considering the contributions made by students during the solution of the problem. It finally stated all the properties and the students tested numbers, applied the algorithms and filled in values in the table.

Suggesting trial and error and later disqualifying it as an appropriate method not only reduced the continuity of the class, but could implicitly suggest to the students that using this approach is not a good way to start a problem (contradicting the structure of the class). It is sufficient to note that a student arrived at the same answer through trial and error as the teacher provided as an exact answer when she applied logarithms ($t = 8.77$). Another aspect related to the richness of processes is that there were some indicators that the teacher did not evaluate, but which are of interest: there was a conversion of records (table - graph), there was reasoning about negative values within the table, the properties of logarithms were presented, and the connection between exponential logarithmic functions was discussed (confusingly). However, all these processes were carried out by the teacher.

[ES4] With respect to representativeness, the teacher indicated that the need to use algorithms to solve the problem was evident among students (used as a technique more than as a concept). She was able to try to solve some logarithms to give a clearer idea of what a logarithm is (perhaps through an arithmetic approach).

Although the concept of a logarithmic function was treated in a manner that was consistent with specific indications of the official curriculum, strictly following this approach sacrifices the opportunity to talk about other situations that could be addressed, such as those mentioned in the methodology of the Program of Study (COSTA RICA, 2012): 1) Using cuttings from magazines, newspapers and statistical data (p 421); 2) History of logarithms (p 426); 3) Sound intensity (p. 426); the Ritcher scale (p 426); 4) Newton's Law of Cooling (p 426); 5) pH measurement (p 427); 6) Compound interest model (p. 427) (the one used by the teacher). Regarding representations, the teacher did present different ways of representing a mathematical object (tabular - symbolic and graphic).

Components and indicators of Cognitive Suitability [CS]

[CS1] previous knowledge. The teacher stated that the first thing she did was to briefly discuss what she explained in the previous class because she considers it to be "always convenient". She stated that students understood the concepts of domain, rank and scope, which was incorrect, based on later evidence. Regarding curricular adaptation to individual differences, she did not carry out reinforcement or expansion activities (although this is of course influenced by her management of time versus the time available). Regarding student learning, there is not enough information to confirm that such learning took place, but it was observed in the video that the teacher works to solve students' difficulties.

[CS3] learning. Regarding logarithms, the teacher stated that "it is not possible to deduce this knowledge. I think you can only show its usefulness." It is possible that there is a relationship between what she thinks about the learning achieved with the fact that there has not been high epistemic suitability. This may be due to the specific indications of the Program of Studies, since they suggest taking the model as true and applying it to a situation that is suggested as real.

[CS4] cognitive demand. The only cognitive achievement the teacher believed that students obtained was being able to read a graph. It should be noted that it was the teacher herself who pointed out that this was prior knowledge.

Components and indicators of interactional suitability [IS]

[IS1] Teacher-student interaction. When the teacher wanted to show how to use the model in an algebraic manner, she repeatedly consulted the students about what they had observed when they worked in groups, which she considers to be valuable in facilitating the inclusion of students since she allowed them to discuss what they had worked on. This positive assessment is debatable, since student comments are based on her answers when they had asked about solutions (in some ways she had suggested the answer and there is no evidence that the students had constructed those concepts themselves).

The teacher indicated that she does not know if the way she ignored students who provided erroneous answers was incorrect, but indicated that she had decided to direct

questions to the persons that she already knew had solved the problem (to obtain the answer she was waiting for) **[IS1a]**.

Despite the filming, the teacher stated out that her class "developed naturally" **[IS1b]**. In addition, she indicated that she had to "deal" with them several times, because the group is particularly undisciplined. She also added that she valued very positively the fact that students are trying to respond even "using an adequate vocabulary" **[IS1c]**. However, she indicated her awareness of the fact that although a large part of the class consisted of lectures, the students do the work that they are assigned **[IS1d]**.

Regarding her way of dealing with the students, she indicated that she considers it to be important that she addressed them in a respectful way and always motivated them to improve, although she added that she considers herself to be a teacher that is "completely dominant and controlling (I always demand attention)" **[IS1e]**.

With regard to the elements that she did not evaluate, it can be stated that her classroom management has lead to very limited discussions on the part of the students, and that she has ignored erroneous arguments (which can be interpreted as evidence of exclusion for those that have participated but have not arrived at the answer that the teacher expects). Another point is that the teacher almost always resolves from the start the difficulties or doubts that students have.

Although she did not consider the point as valuable, she has facilitated interaction among students by having them work in small groups. However, this did not promote autonomy because she provided them with the model to solve the problem. Finally, there is no evidence of having proposed a formative assessment of her class.

Components and indicators of Mediational Suitability [MS]

[MS1] With regard to resources, the teacher concluded that students did not know how to use calculators well, even though the use of calculators was a valuable resource, and that this negatively affected class fluidity. Even so, the teacher instructed students to use calculators as a black box (LINDSAY, 1975) to calculate logarithms, without understanding how the calculation is actually carried out. She also believed that her use of the whiteboard was inadequate (for example, when presenting the table and the graph), and suggested that perhaps she should have used software to present her ideas: "it would have allowed me more flexibility (in showing asymptotes, intersections, and the inverse)" **[MS1a]**.

As for classroom space, there are not too many students or other problems related to physical space. The teacher indicates that she effectively adapted her use of time to the planned activities **[MS1b]**.

Components and indicators of Affective or Emotional Suitability [AS]

[AS2] Attitudes. The teacher does not assess any of the needs, attitudes and emotions of the students, but considers that the fact that students are motivated, and explain their answers and show interest in verifying their work is very positive; regarding this, she

stated that she believes that she has convinced the students of the importance of this topic [AS2]. Although the students do indeed work, this activity in itself does not seem to be especially motivating for the students; on the other hand, the teacher does promote students' interaction and listens to their arguments (even when they are those which she herself previously suggested).

Components and indicators of Ecological Suitability [CIES]

[CIES1] The teacher indicated that she had to modify or adapt the model suggested in the Program of Studies. Her experience working with other groups was that, as formulated, the Program created an obstacle due to the amount of calculations that students had to perform, so she presented the problem with the time data in years. Perhaps the issue that should be considered is that the curricular guidelines were completely followed without a deeper analysis of what could be achieved by following these specific recommendations. With respect to intra- and interdisciplinary connections, the teacher noted that she connected the subject with the exponential function [CIES2], but she had not realized that she left out connections with other disciplines.

Although she did not comment on it, the topic that was being presented seems to have social and labor utility, because it is useful content for this context, although perhaps not at this moment. There is no evidence of any curriculum innovation (since she followed all that is indicated in the curriculum), but the traditional way of teaching was modified in the sense that an effort was made to include student participation.

Although several factors have been presented that could be taken into account in redesign of her class, it should also be noted that the management of the class was carried out responsibly by the teacher, who chose appropriate options when taking into account the amount of time available to introduce this subject. This is consistent with the vision of Huang and Li (2009) discussed in the Introductory section of this document.

Conclusions

In terms of the objective of this research, it was possible to demonstrate that the elements that the teacher uses most frequently to assess her own activity were related to the criteria of epistemic [ES] and interactional suitability [IS], while assessments that appear to a lesser extent can be classified with components and indicators of the didactic suitability criteria [CS], [MS], [AS], [CIES]. This shows, in the first place, that the Onto-semiotic approach to mathematical knowledge and instruction (GODINO; BATANERO; FONT, 2007) represents an appropriate theoretical basis for this type of research, and with the appropriate use of the criteria, components and indicators proposed by Font (2015) for the concept of didactic suitability, it was possible to understand and classify the evaluations made by the teacher.

Regarding epistemic suitability [ES], the teacher was able to detect her error in the definition of the logarithmic function (which was evident when watching the video), but even after reviewing the video several times she did not notice the problems related to the concepts of approach and accuracy. One possible explanation of this finding is

that she assumes that they are part of the common knowledge of students, but it was nonetheless true that this created several problems, including dismissing a correct answer from a student before presenting logarithms. She was likewise unable to identity the most relevant ambiguities that her teaching created.

The most striking aspect of her self-assessment had to do with indicators associated with richness of processes [ES3], since even after the reviews she did not perceive that two aspects were decisive for her class: 1) following the indications of the Program of Studies exactly, and consequently adopting a model as true, stating a problem, and then ensuring that the model completely solves the problem; 2) almost immediately suggesting to the students the method by which they could solve the problem.

In the first case, the way in which the teacher developed the topic did not offer the opportunity to create a model from a real problem; a simpler model that involved exponentials could have been created based on a situation more familiar to the students. Although the specific indications of the Program of Studies (see Table 1) suggest that "[...] to awaken interest and participation, it is proposed to use problems in real contexts that lead to the construction or use of models" (COSTA RICA, 2012, p. 36) and, further, that "[...] the spirit of modeling resides in the identification, manipulation, design and construction of mathematical models about real situations in the environment" (p. 31), it might be necessary to rethink or explore in more depth if the use of an already defined model is a valuable scenario for introducing the logarithmic function concept, or rather if the construction of a simple model could generate or activate other more relevant processes (beyond trial and error, and entering data in a calculator without understanding its meaning). This choice is part of what was pointed out by Mason (2002, 2017) and addressed in the theoretical framework. This situation can also be related to the affective criterion, since it is linked to the beliefs of students towards mathematics, primarily with respect to the way in which mathematics is presented [AS].

With respect to quickly suggesting methods to solve problems, which the teacher did not consider to be an important issue, it should be noted that this totally conditioned the work of students, leading them to use the strategy that the teacher wanted them to use.

With respect to the elements that she evaluated which are related to interactional suitability [IS], it is necessary to note that for the teacher it was explicit and intentional that her class had many of the characteristics of a lecture course. However, a class should involve students carrying out some activities themselves, and the teacher listening to them (in many cases). Being a class of this type, it is difficult for the teacher to make use of incorrect answers by the students (error management), since they are not in a constructive, but rather a highly directed, space. The teacher, as documented on video and in her comments, chose to ignore some incorrect answers, which could be a consequence of the style of the class or an element of interest when considering affective suitability [AS].

Some implications for the development of the teaching function and open questions

The results of the work carried out in this investigation offer tools that may be relevant for teachers who wish to evaluate their own teaching practices. In addition, the methodology and framework of this research could be used as a strategy for systematic

self-assessment of what happens in the classroom. The results of this research agree with other investigations (BREDA; FONT; LIMA, 2015) which confirm that if the teacher manages to create a conceptual framework such as that of didactic suitability, with its components and indicators, through a rubric-type instrument that operationalizes it (such as the one of FONT, 2015), then it could provide not only a strategy or protocol for self-assessment, but also for obtaining elements to guide the search for resources to improve knowledge in mathematics, pedagogy and mathematics didactics, as well as to increase the ability of teachers to identify and reflect on situations in mathematical instruction. It can also serve as support for training of teachers who wish to engage in self-assessment.

This type of self-assessment can be an excellent resource, but it is only a tool that must be evaluated through triangulation using evaluations by peers (as in the case of *Lesson Study*, for example), by supervisors or advisors, or by their own students, which can also provide valuable information about what is happening in the classroom.

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Yuri Morales-López holds a master's degree from the National University, Heredia, Costa Rica, and is a full professor at that university.

Vicenç Font Moll holds a doctorate from the University of Barcelona, and is a full professor at that university.